Eur. Phys. J. B **59**, 93–97 (2007)
DOI: 10.1140/epjb/e2007-00259-3

THE EUROPEAN PHYSICAL JOURNAL B

Dynamical evolution of clustering in complex network of earthquakes

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Received 12 December 2006 / Received in final form 28 June 2007
Published online 5 October 2007 –

Abstract. The network approach plays a distinguished role in contemporary science of complex systems/phenomena. Such an approach has been introduced into seismology in a recent work [S. Abe, N. Suzuki, Europhys. Lett. **65**, 581 (2004)]. Here, we discuss the dynamical property of the earthquake network constructed in California and report the discovery that the values of the clustering coefficient remain stationary before main shocks, suddenly jump up at the main shocks, and then slowly decay following a power law to become stationary again. Thus, the dynamical network approach characterizes main shocks in a peculiar manner.

PACS. 89.75.Da Systems obeying scaling laws – 91.30.-f Seismology – 05.65.+b Self-organized systems

Looking at seismic data from the physics viewpoint, it may be of interest to recognize that it is essentially a field-theoretical system. It consists of the series of a set of values of occurrence time, hypocenter, and magnitude of each earthquake. In other words, seismic moment (its logarithm being magnitude) as a field strength is defined on each discrete spacetime point. However, unlike ordinary field dynamics in physics, both the field strength and spacetime points are inherently random. In spite of such apparent complicatedness, known empirical laws are rather simple. There are in fact two celebrated classical examples. One is the Gutenberg-Richter law [1] for the relationship between frequency and seismic moment. The other is the Omori law [2] for the temporal decay of frequency of aftershocks. Both of them are power laws, indicating complexity/criticality of seismicity.

Instantaneous release of huge energy by a main shock can be thought of as a "quenching" process. The disorder of a complex landscape of the stress distribution at faults in the relevant area is then reorganized by it. Accordingly, a swarm of aftershocks may follow. This process constitutes nonstationary parts of a seismic time series, and, due to the power-law nature of the Omori law, "relaxation" to a stationary state is very slow. In a recent work [3], it has been found that there are striking similarities between the aftershock phenomenon and glassy dynamics, including aging and scaling.

In the previous works [4,5], we have studied the spatio-temporal complexity of seismicity and found that both the spatial distance and time interval between two successive earthquakes obey specific but remarkably simple statistical laws. Those results indicate that successive events are indivisibly correlated, no matter how large their spatial separation is. In fact, there is an investigation [6], which points out that an earthquake can be triggered by a foregoing one, which is more than 1000 km away. This implies that the seismic correlation length may be enormously large, exhibiting a strong similarity to phase transitions and critical phenomena. Accordingly, it is inappropriate to put spatial windows in analysis of seismicity, in general, and a relevant geographical region should be treated in a nonreductionistic manner.

To characterize complexity of event-event correlation in seismicity, we have recently proposed a network approach [7–10], in which seismic data is mapped to a growing stochastic graph. This graph, termed earthquake network, is constructed as follows. A geographical region under consideration is divided into a lot of small cubic cells. A cell is regarded as a vertex of a network if earthquakes with any values of magnitude occurred therein. Two successive events define an edge between two vertices. If they occur in the same cell, a loop is attached to that vertex. The edges efficiently represent event-event correlation mentioned above. The network thus constructed represents dynamical information of seismicity in a peculiar manner. (Another procedure of constructing an earthquake network, which is more complicated than the present one introducing seven parameters including the spatial distance, time interval, magnitude, and so on, is considered for example in Ref. [11].) Several comments on

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this construction are in order. Firstly, it contains a single parameter, the cell size, which determines a scale of coarse graining. Once the cell size is fixed, an earthquake network is unambiguously defined. Since there are no a priori operational rules to determine the cell size, it is of importance to examine the dependence of the property of an earthquake network on it. Secondly, an earthquake network is a directed graph in its nature. Directedness does not bring any difficulties to statistical analysis of connectivity (i.e., degree, the number of edges attached to the vertex under consideration) since, by construction, in-degree and out-degree [12] are identical for each vertex with possible exceptions for the first and last ones in the analysis: that is, in-degree and out-degree do not have to be distinguished each other in the analysis of connectivity. However, directedness becomes essential when the path length (i.e., the number of edges between a pair of connected vertices) and the period (implying after how many subsequent earthquakes the event returns to the initial vertex) are considered, for example. Finally, directedness has to be ignored and the path length should be defined as the smallest value among the possible numbers of edges connecting a pair of vertices, when the small-world nature of an earthquake network is investigated. There, loops have to be removed and multiple edges be replaced by single edges. That is, a full directed earthquake network is reduced to a corresponding simple undirected graph (see Fig. 1 for the schematic description).

An earthquake network and its reduced simple graph constructed in this way are found to be scale-free [7] and of the small world [8], exhibit hierarchical organization and assortative mixing [9], and possess the power-law period distribution [10]. A main reason why an earthquake network is heterogeneous is due to the empirical fact that *aftershocks associated with a main shock tend to return to the locus of the main shock, geographically, and therefore the vertices of main shocks play roles of hubs of the network*.

The network approach has been used in the literature [13] to examine if self-organized-criticality models can reproduce these notable features.

Here, we report a successful application of the dynamical network approach to seismicity. We find through careful analysis that the clustering coefficient exhibits a salient behavior: it is stationary before a main shock, jumps up at the main shock, and then slowly decays as a power law to become stationary again. We ascertain this behavior for some main shocks occurred in 1990's in California. Thus, the dynamical network approach characterizes a main shock in a peculiar manner.

There are several known quantities that can structurally characterize a complex network. Among them, here we consider the clustering coefficient introduced in reference [14]. This quantity is defined for a simple graph, in which there are no loops and multiple edges. A simple graph is conveniently described by the adjacency matrix [15], $A = (a_{ij})$ $(i,\ j = 1,\ 2,\ \ldots,\ N$ with N being the number of vertices contained in the graph). $a_{ii} = 0$, and $a_{ij} = 1$ (0) if the ith and jth vertices are connected

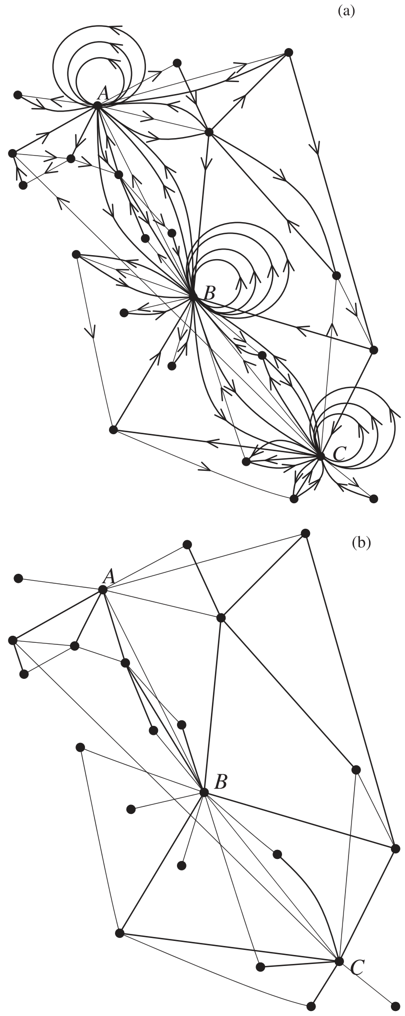

Fig. 1. Schematic descriptions of an earthquake network. (a) A full directed network. The vertices with high values of connectivity, A, B, and C, correspond to main shocks. (b) The simple undirected graph reduced from the full network in (a).

(unconnected) by an edge. The clustering coefficient, C, is then given by

$$C = \frac{1}{N} \sum_{i=1}^{N} c_i, \tag{1}$$

where

$$c_i = \frac{2\, e_i}{k_i(k_i - 1)} \tag{2}$$

with

$$e_i = (A^3)_{ii} \tag{3}$$

and k_i the value of connectivity of the ith vertex. This quantity has the following meaning. Suppose that the ith vertex has k_i neighboring vertices. At most, $k_i(k_i - 1)/2$ edges can exist between them. c_i is the ratio of the actual number of edges of the ith vertex and its neighbors to this maximum value. Thus, it quantifies the degree of adjacency between two vertices neighboring the ith vertex. C is its average over the whole graph. In an earthquake network, c_i quantifies how strongly two aftershocks associated with a main shock (as the ith vertex) are correlated, for example.

Now, we address the question as to how the clustering coefficient changes in time as an earthquake network grows. For this purpose, we have studied the catalog of earthquakes in California, which is available at URL `http://www. data.scec.org/`. In particular, we have focused our attention to three major shocks occurred in 1990's: (a) the Joshua Tree Earthquake (M6.1) at 04:50:23.20 on April 23, 1992, 33°57.60′N latitude, 116°19.02′W longitude, 12.33 km in depth, (b) the Landers Earthquake (M7.3) at 11:57:34.13 on June 28, 1992, 34°12.00′N latitude, 116°26.22′W longitude, 0.97 km in depth, and (c) the Hector Mine Earthquake (M7.1) at 09:46:44.13 on October 16, 1999, 34°35.64′N latitude, 116°16.26′W longitude, 0.02 km in depth. We have taken the intervals of the seismic time series containing these events, divided the intervals into many segments, and constructed the earthquake network of each segment. Then, we have calculated the value of the clustering coefficient of each network. In this way, dynamical evolution of clustering has been explored.

In Figure 2, we present the results on evolution of the clustering coefficient in the case when the length of the segments is fixed to be 240 h long. Here, the cell size 5 km × 5 km × 5 km is examined. A remarkable behavior can be appreciated: the clustering coefficient stays stationary before the main shocks, suddenly jumps up at the moments of the main shocks, and then gradually decays. In the course of decay, some relative maxima appear. A detailed study shows that they are often, but not always, associated with strong aftershocks and are sensitive to the cell size.

To clarify the property of the slow decay in more detail, we present Figure 3, in which shorter-time analysis with 24 h is performed by examining two different cell sizes, 5 km × 5 km × 5 km and 10 km × 10 km × 10 km. As can clearly be appreciated, the "cumulative" clustering coefficient,

$$C(\leqslant n) = \sum_{M=1}^{n} C_M, \tag{4}$$

obeys a definite law, where C_M stands for the clustering coefficient of the network constructed in the interval $24 \times (M-1) \sim 24 \times M$ [hours] after the moment of the main shock at $M = 0$, and $n =$ (hours)/24. Indeed, it is well represented by the following power law:

$$C_M \sim \frac{1}{(1 + M/M_0)^{\alpha}}, \tag{5}$$

where α and M_0 are positive constants, and their values are given in Table 1.

As can be seen in Table 1, the value of M_0 rapidly increases with respect to the cell size. This can be understood as follows. Upon constructing a simple graph, the larger the cell size is, the more loops are removed and multiple edges are replaced by single edges. Accordingly, the simple graph grows slower. This is the reason why the characteristic "time scale", M_0, of evolution becomes larger as the cell size increases.

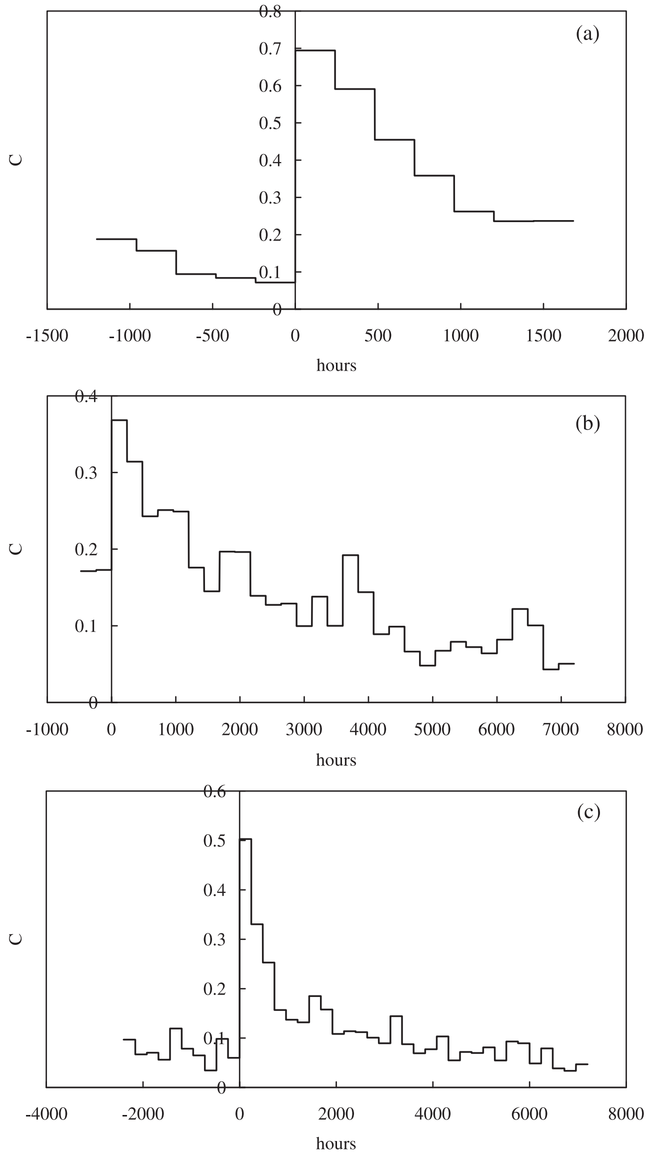

Fig. 2. Evolution of the (dimensionless) clustering coefficient during each 240 h. The origins are adjusted to the moments of the main shocks, that is, (a) the Joshua Tree Earthquake, (b) the Landers Earthquake, and (c) the Hector Mine Earthquake. The numbers of earthquakes, vertices, and simple edges in the time segments in which the clustering coefficient takes its maximum and minimum values are respectively as follows: (a) 3519, 216, and 746 in the segment $240 \times 0 \sim 240 \times 1$, and 546, 210, and 426 in the segment $240 \times 5 \sim 240 \times 6$, (b) 5622, 738, and 3943 in the segment $240 \times 0 \sim 240 \times 1$, and 487, 275, and 463 in the segment $240 \times 28 \sim 240 \times 29$, and (c) 3851, 572, and 2022 in the segment $240 \times 0 \sim 240 \times 1$, and 440, 278, and 413 in the segment $240 \times 28 \sim 240 \times 29$.

We confidently believe that the above discovery is universal, independently of geographical regions to be analyzed. Although we do not present here, the same trend of the clustering coefficient was, in fact, recognized in the analysis of the seismic data taken in Japan.

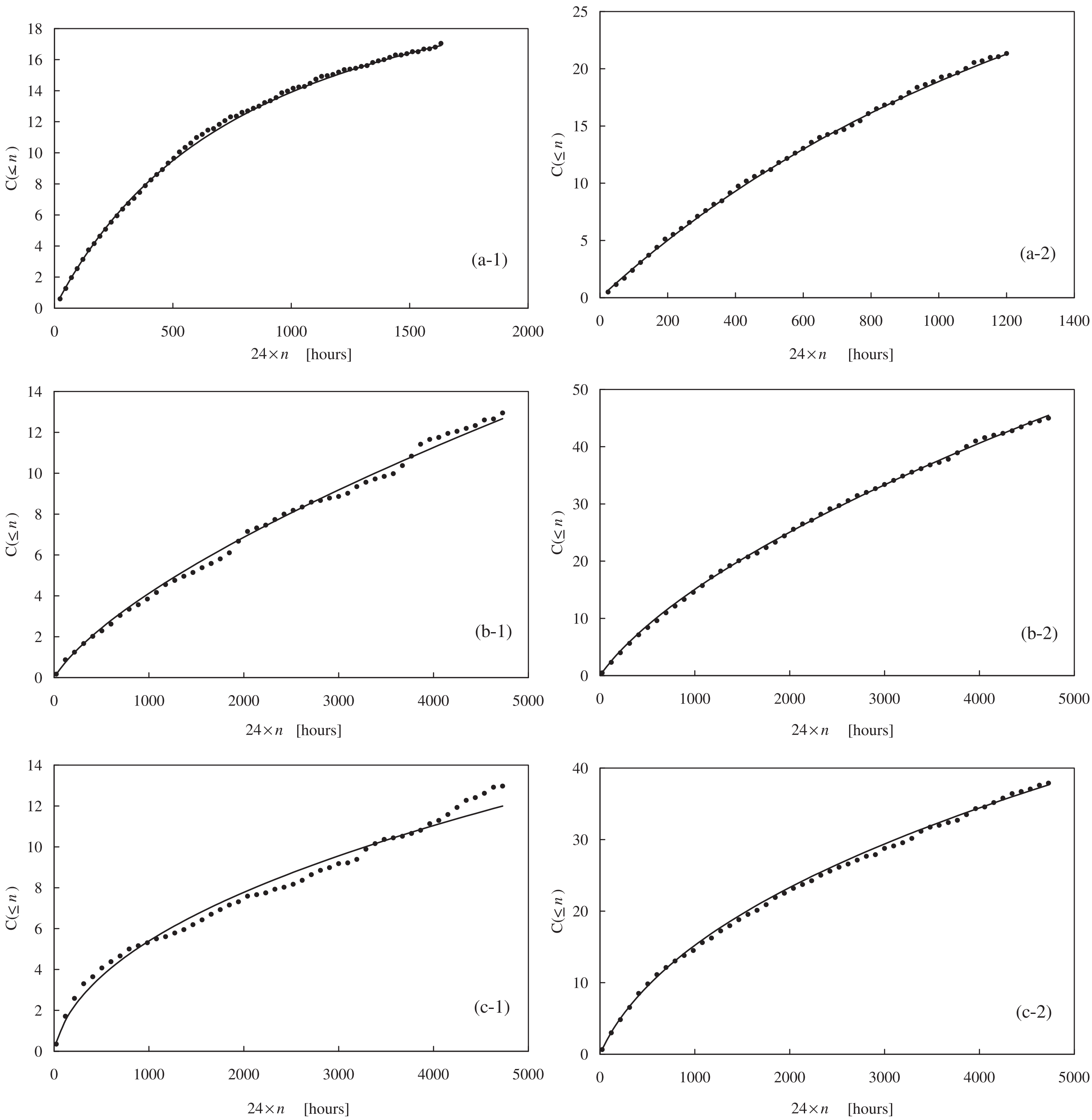

Fig. 3. Evolution of the (dimensionless) cumulative clustering coefficient defined in equation (4) during each 24 h. The solid curves are due to the model in equation (4) with the form in equation (5). (a-1, b-1, c-1) and (a-2, b-2, c-2) are the results for the cell sizes, 5 km × 5 km × 5 km and 10 km × 10 km × 10 km, respectively, for (a) the Joshua Tree Earthquake, (b) the Landers Earthquake, and (c) the Hector Mine Earthquake. The values of the parameters in equation (5) are given in Table 1.

Table 1. The values of the parameters in equation (5) used in Figure 3.

major event	cell size(km × km × km)	$M_0(\times 10^2)$	α
Joshua Tree Earthquake	5	9.84	2.2
	10	48.0	4.0
Landers Earthquake	5	0.618	0.33
	10	1.83	0.40
Hector Mine Earthquake	5	0.122	0.55
	10	1.66	0.59

In conclusion, we have found that the clustering coefficient of the evolving earthquake network remains stationary before a main shock, suddenly jumps up at the main shock, and then slowly decays to become stationary again following the power-law relaxation. In this way, the clustering coefficient is shown to successfully characterize main shocks. We would like to emphasize that the power-law decay after a main shock described in equation (5) might remind one of the Omori law, but actually they are not directly related to each other. This is because, in the definition of the clustering coefficient, loops are removed and multiple edges are replaced by single edges, that is, a number of aftershocks are excluded in the analysis.

One of the authors (S.A.) would like to thank Carmen P. C. Prado for discussions.

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